Math 131B-2: Final Exam Study Sheet

Here is some general information about the final.

- The final is Thursday, June 12th, 11:30-2:30, in Humanities 169 (our usual room).
- I will have office hours Tuesday, June 10th, 1-3 p.m. and Wednesday, June 11th, 10 a.m.-12 p.m.
- The final exam will have ten questions. Eight questions will be variations on the "state a major theorem or definition, then prove, apply, or compute something related to it" format familiar from the first two midterms. Two questions will introduce definitions not previously seen and ask simple questions about how they relate to properties or objects we have already studied.
- Five of the questions on the final will involve the material since the second midterm (Cauchy products, complex numbers, exponential and trigonometric functions, and Fourier theory).
- There will be a question similar to the first question from Midterm 2 ("construct a continuous function or explain why one does not exist.")

Here is a summary of the topics we have covered and the theorems we have proved.

- Metric Spaces
  - Set theory basics; countability.
  - Definition of a metric; properties of open and closed sets; limit points and adherent points; closures and interiors.
  - Completeness; examples of complete and noncomplete metric spaces.
  - Compactness; relationship to limit points; relationship to completeness.
  - Open, closed, and compact sets in $\mathbb{R}^n$; Bolzano-Weierstrass Theorem; Lindelof Covering Theorem; Heine-Borel Theorem.
  - Reading: Apostol 2.2-5, 2.7-8, 2.10-15, 3.13-16, 3.1-2, 3.6-11.
- Continuous Functions
  - Limits of sequences of points in a metric space.
  - Definition(s) of continuous functions of metric spaces.
  - Preimages of open/closed sets under continuous functions are open/closed.
  - Images of compact/connected sets under continuous functions are compact/connected; the Extreme and Intermediate Value Theorems.
  - Uniform continuity.
  - Reading: Apostol 4.1-5, 4.8-13, 4.15-17, 4.19-20.
- Uniform Convergence
  - Definition of uniform convergence; differences between uniform convergence and pointwise convergence; Weierstrass M-test.

  - Uniform convergence and continuity; uniform convergence and integration; uniform convergence and differentiation.
  - The metric space of bounded functions and the d_∞ metric; uniform convergence and d_∞.
  - Interchanging double sums.
  - Reading: Apostol 9.1-6, 9.8, 9.10. 9.12.

- Power Series
  - Uniform convergence of power series; continuity, differentiabilty, and integrability of power series.
  - Abel's Theorem.
  - The Cauchy product and multiplication of power series and real analytic functions.
  - Reading: 9.14-15, 9.18-19, 9.22, 8.24.

- Complex Numbers
  - The complex plane; arithmetic with complex numbers; properties of complex limits; the unit circle.
  - Rigorous definitions and basic properties of e^z, $\sin x$, $\cos x$, e, and π.
  - Reading: Tao Chapter 15, Apostol 1.21-27, 1.29, 1.32.

- Fourier Analysis
  - Periodic functions; the space $C(\mathbb{R}/\mathbb{Z};\mathbb{C})$; definition and properties of the inner product on $C(\mathbb{R}/\mathbb{Z};\mathbb{C})$; the L^2 and L^∞ norms and differences between them.
  - Trigonometric polynomials; definition of the Fourier transform and Fourier coefficients.
  - Definition and properties of periodic convolutions; the Weierstrass approximation theorem for trigonometric polynomials.
  - The Fourier and Plancherel theorems; convergence properties of the Fourier series.
  - Reading: Tao Chapter 16